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Classrooms Socio-Mathematical Discourse: Two Nine-Grade-Dyads' Non-Routine Problem-Solving Engagement

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The purpose of this case study is to explore the complex interplay among student beliefs, problem solving engagement, problem type, and mathematics understanding as well as dynamics within group discourse among four ninth-grade mathematics students. The analysis of both these dyad's 16-week long collaboration reveals that the role of conversation, prolonged problem solving interactions, and on-going negotiations and relationships is key in their transformation. The results suggest, as students developed a culture within their dyads, of problem solving and problem posing, and collaboration, that engagement was increased. After evaluating the various data relating to problem type and participant engagement, it became evident that certain problem types engaged the students more than the others. While it was no surprise that routine problems were not engaging to them, it was also evident that their collaboration and discourse were very different under these circumstances. They were much less likely to question, challenge, argue, negotiate, or probe each other's thinking, and were much more likely to rely on and accept the first answer. There were no efforts to modify, extend, or apply these routine problems to other contexts. The implication of this study for 21st Century classrooms discourse sheds light on the envisioning of curriculum alternatives for mathematics education amidst the many constraints of current and traditional problem solving contexts. This implication for today's technological and information-sensitive classrooms environment is a key for developing learners who are mathematically literate and intellectually autonomous.

Keywords: problem solving, engagement, mathematics, communication, education

Introduction

Historically, problem solving and communication in mathematics has been taught by rote, with very little mindful engagement by the teacher or students. The teacher simply explains the procedures needed to obtain the correct answers, and it makes no difference whether the students work individually or in groups (Ohnemus & Nebraska, 2010, Innes, 2007; Lindquist, 1997; Lindquist et al., 1995). This pervasive manner to problem solving and communication in mathematics undermines the process of meaningful learning; consequently, students regard mathematics as dull and far removed from reality and their own interests. Langer (1997, p. 137) describes a mindful state as an interactive learning participation in which the nature of the interaction "is not a matter of fitting ourselves to an external norm; rather, it is a process by which we give form, meaning, and value to our world". Brown (2007) suggests that curricula and teachers ought to embrace problems, not just their solution. To improve understanding, Hiebert et al. (1996) purport, students must take responsibility for sharing the results of their inquiries and for explaining and justifying their methods. Minimal participation in problem solving, reasoning, and communicative situations frequently results in lack of preparation, performance, and understanding of mathematics. Examining the relationship between achievement gains and the allocation of curriculum resources (both across countries and within countries), the authors argue that "national culture has an impact on learning" (Schmidt et al., 2001, p. 10).

To explore this notion, the authors examined the fundamental aspects of formal education in each country—aspects they believe are affected by social, political, and cultural contexts and are likely to shape student achievement. Schmidt et al. (2001,p. 33) identified countries such as Hong Kong, Korea, and Japan as "high performing," which contrasted with the comparatively low performance level of the United States. Although all countries shared a relatively common core of curriculum, the higher performing countries seemed to utilize more in-depth textbooks and a curriculum that was more organized to take advantage of "the logic of subject matter disciplines"(e.g., mathematics) which "plays an important role in school learning" (Schmidt et al., 2001, p. 356). With this in mind, the authors argue that a set of national priorities

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in content standards can be advantageous and not be equivalent to national control of a system, which is the case in the U.S. For that reason, they propose a reformed curriculum—one that equally addresses both the cognitive demand of tasks and the type of instructional activity. Rather than memorizing inflexible procedures provided by a teacher or textbook, students seem to learn best by constructing their own mathematics. In fact, because construction of knowledge is an essential part of solving problems, the NCTM (2012) placed problem solving at the core of the mathematics curriculum, stressing it in all aspects of mathematics instruction. Problem solving, therefore, should be a part of all mathematics activity, because being mathematically literate means being a good problem solver.

Literature Review

Nonroutine problem solving

Problem solving, as a thinking process, implicates understanding that requires using prior knowledge, concepts, and understandings as well as newly constructed knowledge on the part of the student during his or her own mathematics problem solving experience. Learning mathematics means becoming a mathematical problem solver. In order to understand what nonroutine problem solving is, it is important to understand how problem solving has been treated in the mathematics education literature. From a broader perspective, problem solving involves reaching a goal by providing an answer to a given state in which an answer or solution method is not initially known (NCTM, 2012).

Communications and problem solving process

According to Sfard (2000), learning is inextricably linked to thinking. Learning is thinking and thinking is subordinated to, and informed by, the demands of communication. From this perspective, if having a better understanding of classroom discourse will offer a better understanding of "the dialogue one leads with oneself, then one must realize that investigating communication with others may be the best route to discovering the mechanisms of human thinking" (Sfard 2000, p. 296). To this end, the author claims the best route to discovering human thinking is to investigate the nature of communication. That is, communication within a classroom is not merely helpful; it is integral. Sfard (2000) offers a rather complex aspect of the many components involved in communication. This theory is then utilized in her own goal, which is to study the implications of bringing mathematical objects into being when there is no ready-made discursive focus. In her study, she analyzed a classroom episode in which a group of seventh graders try to solve a problem that was intended for statistical thinking. "The students' exchange is analyzed in terms of the discursive processes that underlie mathematical problem solving and that occasionally bring about the emergence of a new mathematical object" (Sfard, 2000, p. 298).

Student collaboration

Student collaboration, in a dialogic problem-solving process, aids in the development of critical thinking through discussion, clarification of ideas, and evaluation of others' ideas. Stein et al. (1994) pointed out that students' engagement, collaboration, and negotiation through the problem solving process are essential elements of establishing a transformative pedagogy. For example, in a collaborative learning environment, students work together to solve problems just as teams of people work together in the workplace to solve problems.

Student engagement in collaborative learning environments

Student confidence in solving mathematical problems has been found to be a significant predictor of their ability to effect mathematical learning (Pajares & Miller, 1995). This dialogic problem-solving investigation will include learning opportunities in which students are challenged to think critically and to engage collaboratively to resolve their own problems and to understand and use mathematics. When students are allowed to work together in pairs or in a group to negotiate in choosing solution strategies and how to go about resolving their differences, they are given opportunities to collaborate, negotiate, and discuss mathematics as well as work toward the establishment of a supportive and synergistic context of a dialogic community.

Research Design

The purpose of this study was to examine the complex interplay among student beliefs, problem solving engagement, problem type, and mathematics understanding as well as the dynamics within group discourse among four ninth-grade mathematics students. The focus of this study was to explore the following questions:

- What is the relationship between student engagement and problem type?
- How does problem solving discourse evolve as students participate in a collaborative problem solving environment?

The data collection process occurred in two phases. The purpose of the first phase of the study was to gather background information and provide team-building opportunities among and between the volunteer participants.

The first phase lasted approximately six weeks and included the following components: (1) group discussion and negotiation of study procedures with the participants (2) individual participants' completion of questionnaires on Mathematics Learning Inventory (MLI) and one-on-one interviews (3) two, one-hour group meetings for further collection of background information (4) observations of participating students' preliminary pair and group activities starting the third week. During the second phase, the researcher (1) observed and videotaped student dyads during nonroutine mathematics problem-solving interactions (2) followed each dyad videotaped interaction with pair interviews and (3) met with all participants every month for approximately two hours to discuss students' reflections on their nonroutine mathematics problem-solving interactions with peers, problem types, and group discourse.

A final individual exit interview occurred with each of the participating partners at the concluding session of the study. Additionally, videotaped problem-solving sessions were made available for students to review and were used as prompts for follow-up interview sessions. Data from transcriptions of videotaped and audiotaped discussions during problem solving, dyad interviews, and group problem-solving dialogue meetings, as well as students' verbal and written responses to questionnaires and problem-solving journal documentation and reflections were analyzed using a constant comparative method (Bray, Adamson, & Mason, 2007). The emerging categories from the multiple data sources were examined using a matrix of categories for comparing mathematics inquiry, inclusive use of technology, and mathematics understanding as well as changes within the dialogic discourse. This procedural framework was then applied to the selection of participants, research site, etc. This study investigated nonroutine collaborative problem-solving dynamics among four ninth-grade mathematics students. The number of participants provided ample opportunities to observe varying levels of interaction (i.e., one-to-one, within a dyad, and within a group), while still allowing for close observation and analysis permitted in a smaller group setting. The students attended Crossroad Christian School (CCS), Oklahoma, which is a pre-kindergarten through twelfth-grade private school located in an urban area of about 500,000 people. There are several private and state-supported colleges and universities close to this school. While the participants were selected in cooperation with the school, all problem solving sessions took place away from the school either at my house or in public place. As mentioned above in "data collection," a booklet provided by CCS principle is the source for information regarding the school.

Results

The purpose of this study was to examine the complex interplay among student beliefs, mathematics inquiry, and the use of technology as well as mathematics understanding and the evolution of a dialogic community among four ninth-grade mathematics students. The findings of the study present various problem solving situations in which the students were engaged in an effort to track their progression and proficiency in mathematics problem solving, problem posing, communication, use of technical tools, individual inquiry, and conceptual understanding. With that goal in mind, each project was then analyzed in terms of how it related to the research questions that drove this study. These questions are:

-What is the relationship between student engagement and problem type?

-How does problem solving discourse evolve as students participate in a collaborative problem solving environment?

To contribute to the collection of background information, each of the four volunteer participants was asked to complete a survey questionnaire "Math Learning Inventory or (MLI). As these students engaged in collaborate, open-ended, technology supported, dialogic problem solving opportunities, it was hoped they would be able to better reflect upon the potential impact of these experiences. Results of the responses to subsections of MLI follow.

Problem solving approaches

Each participant was asked to reflect on the problem, their initial approaches, and how those techniques fared in execution. After reflection and re-evaluation, the boys were then ready to refine their designs. This was the final stage of this project; the boys (Jim & Bob) were asked to evaluate their projects and make any necessary modifications so that they could test their designs one last time. Jim and Bob had planned two different egg-drop projects, and although they worked together on both of their projects, each was mainly responsible for administrating and executing one project, both of which were successful. They were very familiar with each other's projects and had collaborated in their construction. The other pair, (Clint & Paul), worked individually on their own project and had very little knowledge of each other's work; they seemed to be acting as individuals not as a collaborative pair. Although they were aware of the other's project, they seemed uninterested or in competition with one another.

In fact, during one attempt, Clint broke Paul's egg as a "joke," an act which seemed almost hostile and indicated individuated approaches and goals and a lack of trust. Bob and Jim, however, were very involved with each other's projects and Jim stated that they had asked a friend from geometry class because "a third set of hands was helpful." Bob said they consulted with a neighbor (who was a pilot) in order to make the bottle aerodynamic by adding wings and a tail fin; he believed this would reduce the force of the landing and thereby protect the egg. These two were communicating within their group, but were also involving the community in which they lived. It seemed that their collaborative effort produced more successful attempts than did their counterparts. This initial phase provided the groundwork by which the remainder of the study was conducted. After this initial project was completed, the dyads then moved on to other types of problems, all of which were approached within a supportive, collaborative environment.

Students' engagement on problem type and problem solving discourse

In phase two of this research each participant and dyad were given a variety of nonroutine problems from which they chose and then worked collaboratively. Again, their individual and group progression were evaluated in terms of inquiry, communication, technological support, problem solving development, and mathematics understanding. During the second phase of the study, the researcher (1) observed and videotaped student dyads during their mathematics problem solving engagement (2) followed each dyad videotaped engagement with pair interviews (3) met with all participants once every two weeks for approximately two hours to discuss students' reflections on their problem solving engagement, problem type, and mathematics discourse and (4) negotiated meeting times (for example, whether or not to meet during school holidays such as Thanksgiving, Christmas, and etc.). A final individual exit-interview occurred with each participating partner at the concluding session of the study. The purposes of this interview were to (1) help the researcher to develop sound understandings of student dyads, by allowing the students to re-examine the process and justify their thoughts (2) provide opportunities for students to re-examine their own beliefs with respect to mathematics inquiry, problem type, and mathematics understanding by reflecting on their problem solving interactions and shared experiences. Videotaped problem solving sessions were available for students' review and were used as prompts for follow-up interview sessions. The goals for implementing the second phase of this study were to (1) find what mathematics problems/projects students found interesting and problematic (2) explore the relationship between student engagement and problem type (3) identify emerging patterns with respect to student dyads during their mathematics problem solving engagement and (4) see how problem solving discourse evolve as students participated in a collaborative problem solving environment.

Routine problems

As stated before, these problems were generated from their own Internet search to which the links were provided. These selected routine problems were predominately at an eighth or ninth graded mathematics level; they were consistent with problems that appeared in their textbooks at school (i.e. Pythagorean theorem, right triangles, simple linear equations, etc.), in that they called for the utilization of formulas to solve an unknown. In that manner, these problems were very consistent with traditional belief systems of viewing what mathematics may be and how students should approach solving these problems and, therefore, were very familiar to each participant. It was important to note that the very fact that each participant used the Internet to inquire problems indicated an exploratory use of technology on their part. The majority of the participants expressed a hesitance about using computers and technology in the classroom, but each of them seemed intrigued with the notion of finding their own problems using the Internet; a concept that will be revisited in chapter five. However, when dealing with routine problems, their collective interest seemed to die once the Internet searching and problem collection was finished. Once the problems were gathered from the Internet, each participant tackled to find their answers, in a very traditional, linear fashion, moving very quickly on to the next problem; there seemed to be no challenge involved. One example of routine problems was the Die Problem, where the solver was asked to determine how many times the number two is expected to "come up" if a die is rolled 18 times. From this lack of collaboration, it seems safe to assume that routine problem solving may in fact rob students of opportunities to engage in critical thinking and negotiation. In short, there seemed to have been no synergy involved with these types of mathematical education, which is one of the main reasons reform-minded mathematics educators want to create a supportive environment by utilizing nonroutine problems. Nonroutine problems are much more multi-faceted, and by their nature have the potential to provide meaningful background that may involve association and communication between two or more students in the process of their solution finding. Therefore, the very type of problems used in mathematics curriculum may be relevant to the level of generative collaboration and communication that may lead to deeper conceptual understanding experienced by the students.

Nonroutine problems

Although the participants chose several nonroutine problems to solve collaboratively, the following two examples (the "marble problem," and the "fraction problem") was discussed at length in this session and involved ample opportunities for students' collaboration, perturbation, and argumentation. The two partners posed questions to one another, clarified definitions, tested cases by trial and error, organized data, and experimented with calculators and argumentation. They also engaged in parallel play and attempted to help each other recognize patterns. Such an exchange showed a definitive progression in their communication. This was apparent later when they tackled a logic problem, or the salary problem. The two partners discussed various approaches to solve the problem and both seemed to had benefited from their collaborative problem solving interactions.

Project-based problems

The "project" portion of the study involved a geometric scavenger hunt of sorts at the Omniplex, the egg-drop project, web design, and an Internet treasure hunt. The egg-drop project has already been discussed in detail and is only addressed at this point to make the following observations in terms of discourse evolution and problem solving patterns: During the initial phase, Paul dominated his dyad, and set the pace for their initial planning; he wanted his pair to experiment with Paintbrush and created a preliminary plan before searching the Internet for possible approaches, which contrasted slightly with the other pair. Perhaps because of his prior Internet experience, Jim opted instead to search the Internet and then used Paintbrush to draw a plan. From that point, the differences are much more obvious. Bob and Jim worked together on each project and paid attention to one another during each stage of development, while Clint and Paul were much more individuated. As mentioned before, there was almost a hint of hostility between the two. This may have been due to the fact that Paul's project was much more "polished," and Clint seemed to be jealous of this fact. Similarly, Paul designed several egg-drop executions, while Clint only designed one or two, which showed a difference in their level of engagement. Even though each dyad approached this project differently, one thing was consistent. Both pairs were deeply involved with the project, and experienced a high level of engagement and interest. In fact, during the remainder of the study, they often asked if they could complete a similar project, although they did enjoy the treasure hunt at the Omniplex, where again, the two dyads approached their goal in different manners, although both were highly engaged in the activity.

Both pairs were given hints about the identity of a secret object within the complex, and were then asked to find that object (i.e. Bob and Jim were asked to find something that was brown and stood next to a fence, which turned out to be an elephant, and Clint and Paul were asked to find an object that glinted in the sun, which turned out to be a trashcan). Both pairs employed a notebook for initial thoughts, discussed possible locations with one another, and both were able to find their secret object relatively quickly. Again, Jim and Bob communicated well with one another, but seemed to employ non-verbal communication more so than their counterparts. Additionally, at this level, they seemed to experience more effective communication, but this may be due to the fact (again) of their pre-existing relationship. Before the dyads split up to engage in an Internet treasure hunt, the group as a whole engaged in an Internet-inquiry into no routine problems. Interestingly, as a group, their productivity lagged. For example, Clint passively listened to Paul and nodded his head often (instead of offering verbal communication), while Paul again assumed the role of a teacher as he attempted to explain possible solutions to problems that were found on the Internet. One example of a problem they addressed was another probability problem, which asked the number of times a coin would land either heads or tales. This episode may have been due to the fact that the group had already engaged in previous problem solving that day and had simply lost their enthusiasm for the matter, or it may be indicative of a larger issue. For example, Paul suggested that he was unable to focus in a larger group setting because he could not help but overhear other conversations and solution methods (some of which were incorrect). This suggestion is reminiscent of Sfard and Kieren's (2001) findings that a group setting is not always the most beneficial for students because peer-interactions may actually interfere with an individual's self-talk and the development of potential cognition; that is, he or she may lose a "golden moment" of cognition as a peer attempts to converse while he or she is paying "attention" and is engaged in conversing with self. The other participants agreed that this might be true at times, but that peer-interaction was often helpful in that it allowed them to hear other approaches which had potential to perturb them to re-evaluate their own approaches and methodologies to problem solving and communication. Interestingly, the project that followed this interaction, the Internet-search among dyads, was perhaps the most productive engagement for Clint and Paul. Here, the dyad searched for other problems that they would want to solve, and then attempted to solve various nonroutine problems as a pair. In this process of working on a common goal, the two sat close together, used a calculator, and equally engaged in conversation. They even shared the same piece of paper as they experimented with problem solving; both engaged in self-talk and dialogue. They seemed to have begun to build a community.

Open-ended problems

Each dyad was presented with several open-ended problems to which they did not attempt to solve. These problems were different from any other problems they had seen before. In that, the nature of problems was open-ended. Their solutions required creativity and risks: issues that are seldom, if ever, catered in traditional mathematics education classrooms. Consequently, the pairs talked about these problems, but since they lacked having clear visions or ideas on how to begin or precede their solution strategies, they stopped working on them immediately. This may be indicative of their lack of experiences on such problems, an element which will be addressed in the following chapter. At this point, however, an entry/exit comparison might provide some insight into the dyads' cognitive development, motivation, collaboration, and communication. For that reason, a discussion of the four volunteer participants' exit-interviews which took place at the end of this research study may be helpful.

Conclusion and Discussion

The findings to this research inform a mathematics community interested in incorporating nonroutine problems/projects into current mathematics instruction. The results suggest that as students developed a culture within their dyads, became involved in problem solving and problem posing, and collaboration, their collective and individual engagement were increased. After evaluating the various data relating to problem type and participant engagement, it became evident that certain problem types engaged the students more than the others. While it was no surprise that routine problems were not engaging to them, it was interesting to find that their collaboration and discourse were similarly affected by these mathematics circumstances. The participants were much less likely to question, challenge, argue, negotiate, or probe each others' thinking, and were much more likely to rely on and accept the first answer at which they arrived. There were no efforts to modify, extend, or apply these routine problems to other contexts.

This outcome proves by negation the point Kilpatrick and Silver (2000) made when they concluded that a positive mathematics experience will enable students to apply methodology and knowledge outside the classroom. In this case study, routine problems failed to engage the dyads' cognitive initiative outside a traditional environment and failed to keep them interested in the process of finding a solution. Similarly, these results affirm the thesis presented by Henningsen and Stein (1997): that students decline into procedural thinking when approaching a routine problem that fails to engage creative problem solving. After examining the "big picture," it became evident that over the course of this 16-week period, significant transitional moments existed, during which collaborations among the dyads and the group seemed to change, and the quality of discourse improved for both groups. While not directly related to any specific problem type or context, these transitional moments seemed to be related to on-going negotiations and relationships, as well as to their beliefs and mathematical understandings. Prolonged problem solving and on-going negotiations and collaborations seemed to be related to students' experiences with productive interactions, shared authorities, and meaningful discourse as well as developing a supportive environment that was beneficial to its participants, a conclusion that reiterates the findings of McCaffrey, et al. (2001): a connection in a community setting helps individual cognition and improves mathematics ability and understandings. These conclusions are supported by theorists/researchers who base their studies on the notion that mathematical literacy and understanding include the development of students' autonomy in and out of schools as life-long learners. That is, integrated learning should focus not on accumulation of information, but on mathematical reasoning with a strong emphasis on nonroutine problem solving, problem posing, and understanding, as well as representation and communication of solution findings (Brown, 2007; Davis & Simmt, 2003). In addition, appropriate use of technology, as a problem solving tool, becomes important as students' mathematical competencies develop a point specifically stressed by Cobb (2000), who purports the interest students take when allowed to utilize non-traditional, technical aids and innovations; an interest that is then taken as a shared phenomenon. Likewise, participating in a discourse community, students' collaborative efforts and skills need to be supported in their mathematical experiences. Because problem solving, reasoning, and discussion (Brown, 2007) are the cornerstones of proficiency (Steen, 1999), mathematical literacy and technological competence must include learning opportunities that challenge students to be mindfully engaged (Langer, 1997), to think critically, to use technology collaboratively, and to work on tasks that are worthwhile (Davis et al., 2000).

Implication of the Study

The implication of this study for 21st Century classrooms discourse sheds light on the envisioning of curriculum alternatives for mathematics education amidst the many constraints of current and traditional problem solving contexts. That is, the analysis of both these dyad's long collaboration revealed that the role of conversation, prolonged problem solving interactions, and on-going negotiations and relationships is key in their transformation.

This implication for today's technological and information-sensitive classrooms environment is a key for developing learners who are mathematically literate and intellectually autonomous. Because students' problem solving, reasoning, and discussion is the cornerstone of proficiency, mathematical literacy and technological competence must include learning opportunities that challenge students to be mindfully engaged, to think critically, to use technology collaboratively, and to work on tasks that are worthwhile.

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